INTERNATIONAL BACCALAUREATE

Mathematics: applications and interpretation

MAI

EXERCISES [MAI 1.6]

GEOMETRIC SEQUENCES

Compiled by Christos Nikolaidis

A. Paper 1 questions (SHORT)

1. [Maximum mark: 8]

Consider the geometric sequence $10, 20, 40, 80, \ldots$

(a) Write down the first term u_1 and the common ratio r. [1]

(b) Find the 10^{th} term of the sequence. [2]

(c) Find the sum of the first 10 terms. [2]

(d) Express the general term u_n in terms of n. [1]

(e) **Hence** find the value of n given that $u_n = 20480$ [2]

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2. [Maximum mark: 8]

Consider the geometric sequence $10, 5, 2.5, 1.25, \ldots$

(a) Write down the first term u_1 and the common ratio r. [1]

(b) Find the 10th term of the sequence. [2]

(c) Find the sum of the first 10 terms. [2]

(d) Express the general term u_n in terms of n. [1]

(e) **Hence** find the value of n given that $u_n = 0.3125$ [2]

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3. [Maximum mark: 6]

The first four terms of a sequence are $18, 54, 162, 486$.

(a) Use all four terms to show that this is a geometric sequence. [2]

(b) (i) Find an expression for the n^{th} term of this geometric sequence.

(ii) If the n^{th} term of the sequence is $1062\ 882$, find the value of n. [4]

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4. [Maximum mark: 6]

Let $u_n = 3 \times 2^n$.

(a) Write down the value of u_1, u_2, and u_3. [3]

(b) Express $\sum_{n=1}^{3} \left(3 \times 2^n\right)$ as a sum of three terms and find the result. [2]

(c) Find $\sum_{n=1}^{12} \left(3 \times 2^n\right)$. [1]

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5. [Maximum mark: 6]

Consider the geometric sequence $8, a, 2, \ldots$ for which the common ratio is $\frac{1}{2}$.

(a) Find the value of a. [1]

(b) Find the value of the eighth term. [2]

(c) Find the sum of the first twelve terms. [3]

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6. [Maximum mark: 6]

Consider the geometric sequence $16, 8, a, 2, b, \ldots$

(a) Write down the common ratio. [1]

(b) Write down the value of (i) a; (ii) b. [2]

(c) The sum of the first n terms is 31.9375. Find the value of n. [3]

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7. [Maximum mark: 6]

A geometric sequence has all its terms positive. The 1st term is 7 and the 3rd term is 28.

(a) Find the common ratio. [3]

(b) Find the sum of the first 14 terms. [3]

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8. [Maximum mark: 6]

The seventh term, u_7, of a geometric sequence is 108. The eighth term, u_8, of the sequence is 36.

(a) Write down the common ratio of the sequence. [1]

(b) Find u_1. [2]

The sum of the first k terms in the sequence is $118\,096$.

(c) Find the value of k. [3]

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9. [Maximum mark: 6]

Consider the infinite geometric sequence

$$3000, -1800, 1080, -648, \ldots .$$

(a) Find the common ratio. [2]

(b) Find the 10^{th} term. [2]

(c) Find the sum of the first 10 terms correct to 2 decimal places.

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10. [Maximum mark: 6]

A geometric sequence has a first term of 2 and a common ratio of 1.05.

Find the value of the smallest term that is greater than 500.

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11. [Maximum mark: 8]

The first term of a geometric sequence is 5 while the fourth term is 40. Find

(a) Find the common ratio ***r*.** [2]

(b) Find the fifth term. [1]

(c) Find the sum of the first **ten** terms. [2]

(d) Fins the smallest value of n given that the n-th term exceeds 1000. [2]

(e) Find the first term that exceeds 1000. [1]

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12. [Maximum mark: 8]

The first term of a geometric sequence is 500 while the fourth term is 62.5. Find

(a) Find the common ratio r. [2]

(b) Find the fifth term. [1]

(c) Find the sum of the first **ten** terms correct to 3 decimal places.

(d) Find the smallest value of n given that the n-th term is less than 10. [2]

(e) Find the first term which is less than 10. [1]

13. [Maximum mark: 8]

In a geometric series, $u_1 = \frac{1}{81}$ and $u_4 = \frac{1}{3}$.

(a) Find the value of r. [3]

(b) Find the smallest value of n for which $S_n > 40$. [3]

(c) Confirm the result in (b) by finding the appropriate consecutive sums. [2]

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14. [Maximum mark: 9]

A geometric sequence has second term 12 and fifth term 324.

(a) Calculate the value of the common ratio. [3]

(b) Calculate the 10^{th} term of this sequence. [3]

(c) The k^{th} term is the first term that is greater than 2000. Find the value of k. [3]

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15. [Maximum mark: 9]

A geometric sequence has first term u_1 and common ratio r. Find the values of u_1 and of r in each of the following cases:

(a) if $u_7 = 3645$ and $u_{10} = 98415$ [3]

(b) if $u_7 = 98415$ and $u_{10} = 3645$ [3]

(c) if $S_2 = 20$ and $S_4 = 200$ [3]

16. [Maximum mark: 6]

Let $5,\ x, 45,\ y$ be consecutive terms of a geometric sequence.

(a) Find the possible values of x. [3]

(b) **Hence** find the possible values of y. [3]

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17. [Maximum mark: 8]

Let $k,\ 2k, k+60$ be consecutive terms of a sequence. Find the value of k

(a) if the sequence is arithmetic. [3]

(b) if the sequence is geometric. [3]

(c) Confirm the results (a) and (b). [2]

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18. [Maximum mark: 6]

The three terms $a, 1, b$ are in arithmetic progression. The three terms $1, a, b$ are in geometric progression. Find the value of a and of b given that $a \neq b$.

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19. [Maximum mark: 7]

(a) The first term of an arithmetic sequence is -16 and the eleventh term is 39. Calculate the value of the common difference. [3]

(b) The third term of a geometric sequence is 12 and the fifth term is $\frac{16}{3}$. All the terms in the sequence are positive. Calculate the value of the common ratio. [4]

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20. [Maximum mark: 8]

The first term of an arithmetic sequence is 0 and the common difference is 12.

(a) Find the value of the 96^{th} term of the sequence. [2[

The first term of a geometric sequence is 6. The 6^{th} term of the geometric sequence is equal to the 17^{th} term of the arithmetic sequence given above.

(b) Write down an equation using this information and **hence** calculate the common ratio of the geometric sequence. [3]

The n-th term of the arithmetic sequence is equal to the n -th term of the geometric sequence above.

(c) Find the possible value of n. [3]

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PROBLEMS

21. [Maximum mark: 6]

The annual fees paid to a school for the school years 2000, 2001 and 2002 increase as a geometric progression. The table below shows the fee structure.

Year	Fees (USD)
2000	8000.00
2001	8320.00
2002	8652.80

(a) Calculate the common ratio for the increasing sequence of fees. [2]

In parts (b) and (c) give your answer correct to 2 decimal places.

The fees continue to increase in the same ratio.

(b) Find the fees paid for 2006. [2]

A student attends the school for eight years, starting in 2000.

(c) Find the **total** fees paid for these eight years. [2]

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22. [Maximum mark: 10]

Ann and John go to a swimming pool.

They both swim the first length of the pool in 2 minutes.

The time John takes to swim a length is 6 seconds more than he took to swim the previous length.

The time Ann takes to swim a length is 1.05 times that she took to swim the previous length.

(a) (i) Find the time John takes to swim the third length.

(ii) Show that Ann takes 2.205 minutes to swim the third length. [3]

(b) Find the time taken for Ann to swim a total of 10 lengths of the pool. [3]

(c) Find the time taken for John to swim a total of 10 lengths of the pool. [4]

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23. [Maximum mark: 4]

The tuition fees for the first three years of high school are given in the table below.

Year	Tuition fees (in dollars)
1	2000
2	2500
3	3125

These tuition fees form a geometric sequence.

(a) Find the common ratio, r, for this sequence. [2]

(b) If fees continue to rise at the same rate, calculate (to the nearest dollar) the total cost of tuition fees for the first six years of high school. [2]

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24. [Maximum mark: 5]

The population of Bangor is growing each year. At the end of 1996, the population was 40 000. At the end of 1998, the population was 44 100. Assuming that these annual figures follow a geometric progression, calculate

(a) the population of Bangor at the end of 1997; [2]

(b) the population of Bangor at the end of 1992. [3]

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B. Paper 2 questions (LONG)

25. [Maximum mark: 11]

Portable telephones are first sold in the country *Cellmania* in 1990. During 1990, the number of units sold is 160. In 1991, the number of units sold is 240 and in 1992, the number of units sold is 360.

In 1993 it was noticed that the annual sales formed a geometric sequence with first term 160, the 2nd and 3rd terms being 240 and 360 respectively.

(a) What is the common ratio of this sequence? [1]

Assume that this trend in sales continues.

(b) How many units will be sold during 2002? [3]

(c) In what year does the number of units sold first exceed 5000? [4]

Between 1990 and 1992, the total number of units sold is 760.

(d) What is the total number of units sold between 1990 and 2002? [2]

During this period, the total population of *Cellmania* remains approximately 80 000.

(e) Use this information to suggest a reason why the geometric growth in sales would not continue. [1]

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26. [Maximum mark: 12]

(a) Consider the geometric sequence $-3, 6, -12, 24, \ldots$.

(i) Write down the common ratio.

(ii) Find the 15th term. [3]

Consider the sequence $x - 3, x + 1, 2x + 8, \ldots$.

(b) When $x = 5$, the sequence is geometric.

(i) Write down the first three terms.

(ii) Find the common ratio. [2]

(c) Find the other value of x for which the sequence is geometric. [4]

(d) For this value of x,

(i) write down the first three terms of the geometric sequence.

(ii) find the common ratio of the geometric sequence. [3]

27. [Maximum mark: 10]

A geometric progression G_1 has 1 as its first term and 3 as its common ratio.

(a) The sum of the first n terms of G_1 is $29\,524$. Find n. [3]

A second geometric progression G_2 has the form $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27} \ldots$

(b) State the common ratio for G_2. [1]

(c) Calculate the sum of the first 10 terms of G_2. [2]

(d) Explain why the sum of the first 1000 terms of G_2 will give the same answer as the sum of the first 10 terms, when corrected to three significant figures. [1]

(e) Using your results from parts (a) to (c), or otherwise, calculate the sum of the first 10 terms of the sequence $2, 3\frac{1}{3}, 9\frac{1}{9}, 27\frac{1}{27} \ldots$

Give your answer **correct to one decimal place**. [3]

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28. [Maximum mark: 12]

A geometric sequence has 1024 as its first term and 128 as its fourth term.

(a) Show that the common ratio is $\frac{1}{2}$. [2]

(b) Find the value of the eleventh term. [2]

(c) Find the sum of the first eight terms. [2]

(d) Find the number of terms in the sequence for which the **sum** first exceeds 2047.968. [4]

(e) Confirm the result in (d) by finding the appropriate sums. [2]

29. [Maximum mark: 12]

A National Lottery is offering prizes in a new competition. The winner may choose one of the following.

Option one: $1000 each week for 10 weeks.

Option two: $250 in the first week, $450 in the second week, $650 in the third week, increasing by $200 each week for a total of 10 weeks.

Option three: $10 in the first week, $20 in the second week, $40 in the third week continuing to double for a total of 10 weeks.

(a) Calculate the amount you receive in the tenth week, if you select

(i) **option two**;

(ii) **option three**. [6]

(b) What is the total amount you receive if you select **option two**? [2]

(c) Which option has the greatest total value? Justify your answer by showing all appropriate calculations. [4]

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30. [Maximum mark: 13]

An arithmetic sequence is defined as $u_n = 135 + 7n, \quad n = 1, 2, 3, \ldots$

(a) Calculate u_1, the first term in the sequence. [2]

(b) Show that the common difference is 7. [2]

S_n is the sum of the first n terms of the sequence.

(c) Find an expression for S_n. Give your answer in the form $S_n = An^2 + Bn$, where A and B are constants. [3]

The first term, v_1, of a geometric sequence is 20 and its fourth term v_4 is 67.5.

(d) Show that the common ratio, r, of the geometric sequence is 1.5. [2]

T_n is the sum of the first n terms of the geometric sequence.

(e) Calculate T_7, the sum of the first seven terms of the geometric sequence. [2]

(f) Use your GSC to find the smallest value of n for which $T_n > S_n$. [2]

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